Human capital accumulation may negatively affect economic growth by increasing tax avoidance and reducing effective tax rates and productive public investment. This paper analyzes how this endogenous feedback between human capital accumulation and tax avoidance affects economic growth and macroeconomic dynamics. We found that this interaction have remarkable growth and welfare effects.

**JEL code:** E62, H26, O30, O40, O41.

**Keywords:** Tax avoidance, Tax non compliance, Economic growth.
1. Introduction

Tax evasion and tax avoidance are existing behaviors in all economies.\(^1\) Although both imply paying less amount of taxes, and the economic literature usually denotes both jointly by tax "non-compliance", tax evasion is an illegal activity, while the behavior of tax avoidance is legal. Tax avoidance include not only the use of strategies that allow one to legally minimize tax (for instance increase the superannuation contributions in Australia), but also the search of strategies to exploit deficiencies or uncertainty in the law (known as aggressive tax planning strategies).

Although both phenomena are not a negligible issue for the development and the growth of the economies, only a few number of papers have analyzed the role that the non-compliance tax plays on economic growth.\(^2\) The main conclusion obtained by the literature is that the relation between tax evasion and economic growth is ambiguous, depending mainly on the degree of productivity of public goods. None of these studies analyze tax avoidance explicitly. They either only analyze tax evasion or conflate the two concepts, characterizing both of them as instances of non-compliance.

In this paper, to analyze how non-compliance affects to economic growth we propose to introduce the role of human capital accumulation. It is well known that human capital accumulation is an important source of economic growth because increases the efficiency units of labor. However, there also exists other mechanism through which human capital may reduce economic growth. Our hypothesis is that the relation between non-compliance and human capital accumulation is in both directions. By one hand, non-compliance significatively reduces the government revenues and therefore it affects to the level of public expenditure. In an economy where the human capital accumulation depends on public expenditure, it is clear that non-compliance can also affect this process of human capital accumulation.

By other hand, to avoid taxes is necessary to have some skills, achieved with the

\(^1\)In US, for the period 1976-1992, the nominal tax gap, generated by non-compliance, increases from $ 22.7 billion to $95.3 billion [see Adreoni et al. (1998)]. In New Zealand, Giles (1999) estimated that over the period 1968-94, the total tax gap was the order of 6.4% to 10.2% of total tax liability. More recent estimations for the shadow economy are in Scheneider (2005), although a significant proportion of income is unreported for reasons other than taxation.

\(^2\)Published papers are Roubini and Sala i Martin (1995), Caballe and Panades (1997), Ling and Yang (2001), and Chen (2003). Ho and Yang (2002) and Eichhorn (2004a, b) have also manuscripts on this issue.
educational level. The results reported for the relation between the taxpayer’s educational level and the avoidance and aggressive tax planning behavior are undoubted. Auerbach et al (2002) tested that tax avoidance increases over time because taxpayers learned successful techniques to shelter gains from taxes. Fox and Luna (2005) find that the number of limited liability companies has a positive relationship with the percentage of the population with bachelor degrees. Murphy (2006) finds that the taxpayers involved in aggressive tax planning are considerably more educated than taxpayers from general population. However, when tax evasion behavior is analyzed the obtained results are not conclusive.

According to these empirical evidences, to introduce the role of the human capital in the debate of how non-compliance affects to economic growth, requires to separating explicitly tax evasion and tax avoidance. Since the relation between human capital and tax avoidance seems to be clearer, although the economic growth literature has been focused more on tax evasion, in this paper we will consider explicitly tax avoidance. To highlight the relevance of the tax avoidance problem, Oxfam (2000) have computed that the cost of corporate tax avoidance to developing countries is around $50 billion annually. Moreover, Braithwaite (2003) relates that a multitude of strategies that seek to exploit deficiencies in the law are continuously being devised each year. Murphy (2002) also shows that during the 1990s, an estimated $4 billion in tax revenue was lost as a result of 42,000 Australians becoming involved in aggressive mass marketed tax schemes.

Summarizing, human capital accumulation increases tax avoidance and, therefore reduces effective tax rate and productive public investment, which determines the future human capital. The aim of this paper is to analyze how this endogenous feedback between human capital accumulation and tax avoidance affects economic growth and macroeconomic dynamics. To do that, we will introduce endogenous tax avoidance in an endogenous growth model with human and public capital accumulation. The analysis will show that the interaction between human capital accumulation and tax avoidance may have remarkable growth and welfare effects. Moreover, we will show how these two effects are

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3One can also consider that higher your income is, higher your possibilities to pay someone who tell you how avoid taxes. That issue does not invalid our statement.

4Some papers find that more education reduces the preference for evasion attitude, [Kinsey and Grasmick (1993) and Hite (1997)]. However, others have found mixed results [Jackson and Milliron (1986)]

5Another argument in favor to focus on tax avoidance instead of tax evasion, is that a more realistic behavior of the taxpayers in response to tax increases is the avoidance, which not imply an illegal activity.
in general of opposite sign. Avoidance can either increase or reduce economic growth depending on the value of the legal tax rate and on the intensity of the tax avoidance technology.

The paper is organized as follows. Section 2 presents the economic model. Section 3 defines the balanced growth equilibrium of the economy. Section 4 numerically characterizes the growth and dynamics effects of human capital accumulation, fiscal policy and avoidance. Finally, Section 5 summarizes the main findings of our analysis and prospects for the direction of future research.

2. The economy

We consider an infinite horizon, continuous time, endogenous growth model with accumulation of private and public capital. In particular, we extend the one-sector growth model with productive public investment introduced by Barro (1990). We introduce two main modifications. First, we consider public capital instead of public expenditure in the line of Futagami et al. (1993). Second, the effective tax rate is endogenous because of the tax avoidance.

Our economy consists of competitive firms, a representative household and the government. We assume that the unique good of this economy is produced by means of a production function that uses private and public capital as inputs. We consider a broad definition of private capital to include physical and human capital. For simplicity in the exposition, from now on we will refer to human capital to denote this broad stock of capital. We consider a Cobb-Douglas production function, so that output is given by

\[ y_t = Ah_t^\beta g_t^{1-\beta}, \]  

with \( \beta \in (0, 1) \) and where \( A \) is the constant total factor productivity; \( h_t \) is the per capita stock of human capital; and \( g_t \) is the per capita stock of public capital. Observe that the production function exhibits private diminishing returns to human capital, and social constant returns to scale. This implies that the competitive firms operate with strictly positive profits.\(^6\) In particular, profit maximization implies that the rental price of human capital equals its marginal

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\(^6\)We can also interpret profits as the return of a fixed input. For instance, we can consider that the production function uses raw labor as an input that is exogenously supplied by the household. In this case we could follow Mankiw et al. (1992) to assume that labor and human capital cannot be disentangled, but they exhibit different marginal productivities.
productivity:

\[ w_t = \beta A h_t^{\beta - 1} g_t^{1-\beta}, \quad (2.2) \]

and profits are given by

\[ \pi_t = (1 - \beta) A h_t^{\beta} g_t^{1-\beta}. \quad (2.3) \]

Output \( y_t \) can either be used for consumption \( c_t \), producing new human capital or public investment \( I_t \). Hence, the stock of human capital evolves as

\[ \dot{h}_t = y_t - c_t - I_t - \delta h_t, \quad (2.4) \]

where \( \delta \in (0,1) \) is the depreciation rate of human capital stock.

The household’s preferences are represented by the discounted lifetime utility:

\[ U_t = \int_0^\infty \left( \frac{c_t^{1-\sigma} - 1}{1 - \sigma} \right) e^{-\rho t} dt, \quad (2.5) \]

where \( \rho > 0 \) is the constant subjective rate of time preference, and \( \sigma > 0 \) denotes the inverse of the constant elasticity of intertemporal substitution. Household is endowed with private capital that inelastically supplies to firms. She allocates her after-tax income to consumption and investment in human capital. Accordingly, the consumer’s budget constraint is given by

\[ (w_t h_t + \pi_t) [1 - \tau (1 - \phi_t)] = c_t + \dot{h}_t + \delta h_t, \quad (2.6) \]

where \( \tau \in (0,1) \) is the tax rate on total income, and \( \phi_t \equiv \Phi \left( \frac{h_t}{y_t} \right) \in (0,1) \) is the rate of tax avoidance. Given a tax rate \( \tau \) set by the government, the household faces to an effective tax rate given by \( 1 - \tau (1 - \phi_t) \). We assume that the ability for avoiding taxes is an increasing and concave function of the ratio from human capital to output: \( \Phi' > 0 \) and \( \Phi'' < 0 \). This modelling assumption eliminates the effects of sustained growth on tax avoidance. If tax avoidance would not be only a function of human capital-output ratio, then the level of avoidance will explode as the stock of human capital tends to infinity. In other words, this assumption is necessary to ensure the existence of a balanced growth path along which output grows at a constant rate. Tax avoidance depends negatively of output. For a given level of human capital, the sources of income become more complex as the economy develops and, therefore, tax avoidance is more complicate. We will consider the following functional form for the rate of tax avoidance:

\[ \phi_t \equiv \Phi \left( \frac{h_t}{y_t} \right) = de^{-\frac{h_t}{y_t}}, \quad (2.7) \]
with $d \in (0, 1)$ represents the maximum rate of avoidance, so that this parameter measures the avoidance intensity or, equivalently, the productivity of human capital in avoiding taxes. Finally, observe that tax avoidance is a non-rival activity, i.e., household immediately reduces the effective tax rate when she acquires new human capital.\footnote{Alternatively, we may have considered that individuals should allocate their stock of human capital either to produce or to avoid taxes. This would be a natural extension of the present analysis.}

The objective of the household is to maximize the utility function (2.5) subject to (2.6) and (2.7). From the first-order conditions of this maximization problem, we obtain that the household’s optimal plan is given by:

$$\frac{\dot{c}_{t}}{c_{t}} = \left( \frac{1}{\sigma} \right) \left\{ w_{t} [1 - \tau (1 - \phi_{t})] + \tau \left( \frac{\partial \phi_{t}}{\partial h_{t}} \right) (w_{t} h_{t} + \pi_{t}) - \rho - \delta \right\}, \quad (2.8)$$

together with the budget constraint (2.6), the avoidance rate (2.7) and the usual transversality condition

$$\lim_{t \to \infty} e^{-\rho t} c_{t}^{-\sigma} h_{t}. \quad (2.9)$$

Equation (2.8) is the Euler equation that determines the intertemporal allocation of consumption and investment, i.e., the growth rate of consumption. As usual, this condition equates the return from investing one unit of output $y_{t}$ and the growth of the marginal utility arising from consuming one additional unit of this good. Because of endogenous avoidance, in our economy the marginal return from investing in human capital has two components. The first component is the market return given by the effective after-tax wage rate:

$$R_{t}^{1} = w_{t} [1 - \tau (1 - \phi_{t})]. \quad (2.10)$$

In addition, investing in human capital in our economy also increases avoidance, which reduces the effective tax rate and thus this increases the disposable income:

$$R_{t}^{2} = \tau \left( \frac{\partial \phi_{t}}{\partial h_{t}} \right) (w_{t} h_{t} + \pi_{t}). \quad (2.11)$$

The government in this economy only provides productive public capital to firms. This government finances public investment $I_{t}$ by means of a flat-tax income tax. We assume that this public intervention is subject to a balanced budget. Tax
revenues depends on the legal tax rate \( \tau \) and on the rate of avoidance \( \phi_t \). Hence, public investment is given by

\[
I_t = \tau (1 - \phi_t) (w_t h_t + \pi_t)
\]

Finally, the law of motion for public capital is

\[
\dot{g}_t = I_t - \eta g_t,
\]

where \( \eta \in (0, 1) \) is the depreciation rate of public capital.

3. Competitive equilibrium

Given the initial stocks of human capital \( h_0 \) and public capital \( g_0 \), a competitive equilibrium under a fiscal policy \( \tau \) is defined as the time path of prices \( \{w_t\} \) and quantities \( \{c_t, k_t, g_t, \pi_t\} \) that satisfies: (i) the utility maximization conditions (2.8), (2.6), (2.7) and (2.9); (ii) the profit maximization conditions (2.2) and (2.3); and (iii) the government constraints (2.12) and (2.13). After manipulating these equilibrium conditions we obtain the growth rate of human capital

\[
\frac{h_t}{h_t} = \left[ 1 - \tau \left(1 - de^{-A(\frac{h_t}{g_t})^\beta - 1}\right) \right] A \left( \frac{h_t}{g_t} \right)^{\beta - 1} - \frac{c_t}{h_t} - \delta,
\]

of public capital

\[
\frac{g_t}{g_t} = \tau \left(1 - de^{-A(\frac{h_t}{g_t})^\beta - 1}\right) A \left( \frac{h_t}{g_t} \right)^\beta - \eta,
\]

and of consumption

\[
\frac{c_t}{c_t} = \left( \frac{1}{\sigma} \right) \left\{ \beta \left[ 1 - \tau \left(1 - de^{-A(\frac{h_t}{g_t})^\beta - 1}\right) \right] A \left( \frac{h_t}{g_t} \right)^{\beta - 1} \right. \\
+ \left. \tau de^{-A(\frac{h_t}{g_t})^\beta - 1} A^2 \left( \frac{h_t}{g_t} \right)^2 - \rho - \delta \right\}.
\]

Our economy exhibits a balanced growth path (BGP, henceforth) equilibrium, along which the stock of human capital, consumption and the stock of public capital grow at a constant and equal rate denoted by \( \gamma \), whereas the rental rate of human capital and the output-human capital ratio remain constant.
As is standard procedure, to proceed with the analysis, we consider the aggregate ratios \( \kappa \tau = \eta \tau \gamma \tau \) and \( \lambda \tau = \chi \tau \eta \tau \gamma \tau \), which will be constant along a BGP. Combining (3.1) and (3.2), we get

\[
\frac{\dot{z}_t}{z_t} = \left[ 1 - \tau \left( 1 - de^{-Az_t^{\beta-1}} \right) \right] (1 + z_t) A z_t^{\beta-1} - x_t + \eta - \delta, \tag{3.4}
\]

and combining (3.1) and (3.3) we obtain

\[
\frac{\dot{x}_t}{x_t} = \left( \frac{Az_t^{\beta-1}}{\sigma} \right) \left\{ (\beta - \sigma) \left[ 1 - \tau \left( 1 - de^{-Az_t^{\beta-1}} \right) \right] + \tau de^{-Az_t^{\beta-1}} A z_t^{(\beta-1)} \right\} + x_t - \frac{\delta (1 - \sigma) + \rho}{\sigma}. \tag{3.5}
\]

The dynamic equilibrium is thus fully characterized by a set of path \( \{ z_t, x_t \} \) such that, given the initial value \( z_0 \) of human to public capital ratio, solves the system of equations (3.4) and (3.5), and satisfies the transversality condition (2.9). Observe that in this characterization of the equilibrium paths \( z_t \) is the unique state variable and \( x_t \) is the control variable.

4. Numerical analysis

It is not possible to analytically prove the existence, uniqueness and stability properties of the BGP equilibrium. The existence of an avoidance function that depends on \( z_t \) impedes the analytical characterization of these properties. Furthermore, note that, unlike Barro (1990), this is not an AK model. Both the existence of public capital and of an endogenous rate of avoidance generates transitional dynamics. Hence, our economy exhibits transitional adjustment when there are initial imbalances in human and public capital. In the rest of the paper we will perform numerical simulations to characterize the growth and dynamics effects of human capital accumulation, fiscal policy and avoidance.

4.1. Calibration

We set the parameter values of our economy by mapping its BGP equilibrium onto some facts observed in the data of US economy. This defines our benchmark economy from which we numerically characterize the effects of avoidance and fiscal policy on the long-run growth rate and welfare. In performing this calibration exercises we should note that we are considering that \( h_t \) is a broad measure of capital that includes physical and human capital. Hence, in this exercises
we have to take into account this fact in fitting the model with the data. The calibration target that we use are the following: (i) the private capital share is taken from Mankiw et al. (1992); (ii) a private investment-capital ratio equal to 0.076; (iii) a stationary growth rate of 2%; (iv) an after-tax net marginal return on human capital equal to 5.6%; (v) a public capital to GDP ratio of 2; (vi) a public investment to GDP ratio equal to 0.05; (vii) a intertemporal elasticity of substitution of 2; and (viii) an avoidance rate of 6%. There are not disposable estimations on the length of avoidance. However, as a benchmark value we take this rate of avoidance, which seems to correspond with a conservative approximation of the actual value. We summarize the parameters of our benchmark economy in Table 1. Note that the benchmark tax rate in this model is equal to 5.26%, which corresponds with an effective rate of 5%. However, in our economy the only public expenditure is the public investment, and the public budget is balanced.

4.2. Growth effects

Taking the benchmark economy as a starting point, we have computed the stationary growth rate for different values of the legal tax rate $\tau$ and of the avoidance intensity $d$. Table 2 shows the results of these simulations. We derive two main conclusions from these results. By reading the table by rows, we first observe that the stationary growth rate $\gamma$ decreases with the avoidance intensity $d$ when the tax rate $\tau$ is sufficiently small. By the contrary, when the tax rate $\tau$ is sufficiently high, the relationship between the long-run growth rate $\gamma$ and the avoidance intensity $d$ has inverted-U shape. In particular, we obtain in our simulations that the threshold value of $\tau$ that modifies the pattern in the growth effects of avoidance intensity is equal to 0.26. The second important conclusion from Table 2 is that the steady-state growth rate $\gamma$ is a function with an inverted-U shape of the tax rate $\tau$. For a given value of avoidance intensity $d$, the stationary growth rate $\gamma$ increases with $\tau$ until reaching a maximum, and then that growth rate decreases for larger values of $\tau$.

We first summarize the growth effects of avoidance in the following result:
**Result 1.** There exists a threshold value $\bar{\tau}$ of tax rate, such that

(a) If $\tau < \bar{\tau}$, then $\frac{\partial \gamma}{\partial \delta} < 0$;

(b) If $\tau > \bar{\tau}$, then there is a growth-maximizing level of $d$ and, moreover, this level is increasing in $\tau$.

From this result we conclude that tax avoidance can either stimulate or reduce the long-run economic growth depending on the value of legal tax rate $\tau$ and the intensity of tax avoidance $d$. The first panel of Figure 1 illustrates this conclusion by plotting the relationship between the stationary growth rate $\gamma$ and the avoidance intensity $d$ for two alternatives values of the legal tax rates: (i) $\tau = 0.1$ (continuous line); and (ii) $\tau = 0.4$ (dashed line). The growth rate has a negative slope for all values of avoidance intensity $d$ when $\tau = 0.1$, whereas that rate reaches a maximum value at some value of $d$ in $(0, 1)$ when $\tau = 0.4$. Hence, avoidance may be positive for growth when the tax rate takes sufficiently high values. This conclusion leads us to compute the growth-maximizing value of avoidance rate for each value of the tax rate. The results of this exercise are given by the second panel of Figure 1 and by Table 3. Observe that stationary growth is maximized in absence of avoidance ($d = 0$) if $\tau$ is smaller than $\bar{\tau} = 0.26$, whereas when $\tau > \bar{\tau} = 0.26$ the growth-maximizing value of $d$ is strictly-positive and increasing with $\tau$. Table 3 computes the growth-maximizing value of $d$ for alternative values of $\tau$ (second column), as well as the corresponding effective tax-rate (third column), the stationary growth rate (forth column), and the deviation of these maximum growth rates from the benchmark value of 2% (fifth column). We observe that the growth rate is much larger (the double in average) under the growth-maximizing value of avoidance intensity $d$ than under its benchmark value. Therefore, we can conclude that the growth effects of avoidance are important and not trivial.

[Insert Figure 1 and Table 3]

The intuition behind Result 1 is simple. The growth effects of avoidance come from the distortion of the effective tax rate on the accumulation of human capital. Remember that the marginal return from accumulating human capital has two components: (i) the effective after-tax wage rate ($R_1$); and (ii) the increase in the avoidance and thus in the disposable income ($R_2$). We must characterize the effects of avoidance on these two returns from investing in human capital. Consider an
increase in the intensity $d$ of avoidance. The channels through which this shock affects long-run growth can be summarized as follows:

(i) The associated reduction in the effective tax rate has two ambiguous effects on the effective after-tax wage rate ($R_1$). First, the disposable income goes up. Second, this change stimulates capital accumulation, which will reduce the marginal productivity of human capital. This productivity effect dominates when the effective tax rate is low (small values of $\tau$).

(ii) In addition the increase in $d$ also affects long-run growth by raising the avoidance gain from investment ($R_2$). The smaller the effective tax rate, the smaller are this effect on the avoidance consequences of investing in human capital. In any case, this avoidance effect seems to reinforce the disposable income effect described above.

By using the numerical results on Table 1, we next summarize the growth effects of the tax rate $\tau$ in the following result:

**Result 2.** There exists a threshold value $\tau^*$ of tax rate, such that

(a) If $\tau < \tau^*$, then $\frac{\partial \gamma}{\partial \tau} > 0$;

(b) If $\tau > \tau^*$, then $\frac{\partial \gamma}{\partial \tau} < 0$.

Moreover, $\tau^* > 1 - \beta$, and $\frac{\partial \gamma^*}{\partial d} > 0$.

From this result we conclude that the threshold $\tau^*$ is the value of tax rate that maximizes the long-run economic growth. More interesting, this growth-maximizing tax rate is increasing in the intensity of avoidance $d$. Therefore, this tax rate $\tau^*$ is larger than the elasticity $1 - \beta$ of output $y_t$ with respect to public capital $g_t$ provided that avoidance is strictly positive ($d > 0$). Obviously, in absence of avoidance ($d = 0$) we obtain that $\tau^* = 1 - \beta$ as was established by Barro (1990) and Futagami et al. (1993). Figure 2 and Table 4 clearly illustrate these conclusions. The first panel shows the dependence of growth rate $\gamma$ on the tax rate $\tau$ for the benchmark value of avoidance intensity $d$. This dependence has a inverted-U shape, so that there exists a interior value of $\tau$ that maximizes the stationary growth rate. The second panel of Figure 2 and Table 4 compute the growth-maximizing tax rate $\tau^*$ as a function of the avoidance intensity $d$. This tax rate is increasing in $d$. Furthermore, the growth rate is much larger (more
than the double in average) under the growth-maximizing tax rate than under the benchmark tax rate. These results corroborate the importance of avoidance for the long-run growth rate.

[Insert Figure 2 and Table 4]

The intuition behind Result 2 is easily obtained by checking to the distortion on the decision of accumulating human capital. Consider an increase in the legal tax rate $\tau$. The channels through which this policy change affects long-run growth can be summarized as follows:

(i) As in Barro (1990), the associated increase in the effective tax rate has two ambiguous effects on the effective after-tax wage rate ($R_1$). First, the disposable income goes down. Second, this change discourages the accumulation of human capital, which will drive the marginal productivity of human capital up. This productivity effect dominates when the effective tax rate is low (small values of $\tau$).

(ii) In addition the increase in $\tau$ also affects growth by raising the avoidance gain from accumulating human capital ($R_2$). This avoidance effect of tax rate reinforces the positive productivity effect of increasing $\tau$. Therefore, this avoidance effect ($\Delta R_2$) increases the growth-maximizing tax rate above the elasticity $1 - \beta$ of output with respect to public capital.

Before closing this subsection, we study how the elasticity $1 - \beta$ of output with respect of public capital affects the derived conclusions. It is clear that the contribution of public capital to production is a crucial piece of the mechanism that we have here proposed to explain the relationship between avoidance, human capital accumulation and growth. We now perform some sensitivity analysis with respect to this elasticity. Table 5 illustrates the dependence of the growth-maximizing value of avoidance intensity $d$ with respect to $1 - \beta$. Observe that the growth effects of avoidance are qualitatively robust to the value of $1 - \beta$. Given a value of $\tau$, the growth-maximizing value of $d$ increases when $1 - \beta$ goes to the extreme values in its domain $(0, 1)$. By the contrary, Table 6 shows how the growth-maximizing tax rate depends on the elasticity of output with respect to public capital. The growth effects of legal tax rate $\tau$ are also qualitatively robust to the value of $1 - \beta$. Given a value of avoidance intensity $d$, the growth-maximizing value of $\tau$ is in general decreasing in $1 - \beta$.

[Insert Tables 5 and 6]
4.3. Welfare effects

In this subsection we characterize the dynamic adjustment of our economy to imbalances between human and public capital, and how this adjustment depends on the intensity of avoidance $d$. In particular, we study the dynamic response of the economy to a negative shock on the stock of human capital $h_t$ and to a variation on the legal tax rate $\tau$. The procedure of our analysis is the following. We assume that the economy is initially in the benchmark BGP and, unexpectedly, one of the proposed perturbations is introduced in a permanent basis. We characterize the dynamic adjustment to the new BGP by computing the associated equilibrium paths of the aggregate variables.

To illustrate the effects of avoidance intensity on the dynamic response we compute the welfare cost of the aforementioned exogenous shocks. As in Lucas (1987), we measure the welfare cost by the percentage increase in consumption that the household should receive as a compensation for the shock. To illustrate this procedure, we denote the policy function relating the equilibrium value of consumption $c_t$ with the capital ratio $z_t$ by $c_t = c(z_t; \theta)$, where $\theta = \{A, \beta, \delta, \eta, \rho, \sigma, \tau, d, z_0\}$ is the vector of fundamentals. Consider that the vector of fundamental change from $\theta_0$, corresponding to the benchmark economy, to $\theta_1$. The welfare cost of this change is the constant fraction $\lambda$ of consumption that one should give to the household every period after the shock to obtain the same utility as in the situation where the economy permanently stays in the benchmark BGP. Thus, the fraction $\lambda$ is the solution of the following equation:

$$
\int_0^\infty \left[ \frac{c(z^*; \theta_0) - 1}{1 - \sigma} \right] e^{\rho t} dt = \int_0^\infty \left[ \frac{c(z_t; \theta_1)(1 + \lambda) - 1}{1 - \sigma} \right] e^{\rho t} dt,
$$

where $z^*$ denotes the stationary value of capital ratio $z_t$ along the benchmark BGP. If $\lambda$ is positive (negative), then the shock generates a welfare cost (gain) because this means that household should receive (give) consumption as a compensation for the shock. We are interesting in numerically studying how our measure of welfare cost $\lambda$ depends on the avoidance intensity $d$. In order to make the welfare costs comparable across the alternatives values of $d$, we will adjust the TFP parameter $A$ when we change $d$. This ensures that all of the simulated economies converge to the same stationary growth rate regarding their different avoidance intensity.

Figures 3 and 4 illustrate the welfare costs of reducing the stock of human capital $h_0$ by a 15%. The main conclusion is that the effects of avoidance on this welfare cost depends on the legal tax rate $\tau$, although they are small and opposite
of the growth effects of avoidance. Figure 3 compute this welfare cost when the tax rate takes its benchmark value, whereas Figure 4 compute the this cost for a $\tau = 0.4$. When $\tau$ is in the benchmark level, the welfare cost is decreasing in the intensity of avoidance $d$. However, the magnitude of the effects of $d$ on the welfare cost in that case are very small. To better illustrate this point, the second panel of Figure 3 shows the logarithmic deviation of the welfare cost under each value of $d$ with respect to the welfare cost in absence of avoidance ($d = 0$). We confirm that avoidance reduces the welfare cost when $\tau = 0.0526$ (benchmark), although the maximum reduction is of 12% for very high values of $d$. If $d$ is smaller than 0.7, then the deviation of welfare cost are smaller than 2%.

Figure 4 shows that when $\tau = 0.4$ the relationship between the welfare cost of the negative shock in $h_0$ and the intensity of avoidance $d$ is not monotonic. The welfare cost is increasing (decreasing) in $d$ for sufficiently small (large) values of this parameter. In this case, there then exists an interior valor of $d$ in $(0, 1)$ where the welfare cost of the negative shock in human capital reaches a maximum value. In any case, the welfare cost is again of the quite small magnitude. The second panel of Figure 4 shows that the maximum logarithmic deviation from the situation without avoidance ($d = 0$) is smaller than 4%. Furthermore, the deviation is around 2% at the value of $d$ given the maximum welfare cost.

Figure 5 presents the welfare costs of reducing the legal tax rate $\tau$ from its benchmark value to 0.04. The main conclusions is that this tax reduction generates a welfare cost. The intuition behind this result is simple. The policy change increases disposable income, and stimulates the accumulation of human capital, which drives the marginal productivity of human capital down. Furthermore, the reduction in $\tau$ decreases the effect of investing on the ability for avoiding taxes ($R_2$). This reinforces the aforementioned effect from the reduction in the marginal productivity of capital. The first panel of Figure 5 shows that this welfare cost is increasing in the intensity of avoidance $d$. Moreover, the second panel illustrates that the effects of avoidance on the welfare cost of the considered policy reform is quite large. The logarithmic deviation of welfare cost from the welfare cost in absence of avoidance ($d = 0$) is between 0 and 50% depending on the value of the avoidance intensity $d$. 

[Insert Figures 3 and 4]

[Insert Figure 5]
As was explained above, the welfare cost of reducing the tax rate from its benchmark value derives from the fact that this value is quite below the social optimal value. We have checked that if the initial value of the tax rate is sufficiently large, the results are just the opposite of those provided by Figure 5. In that case, reducing the tax rate generates a welfare gain, whereas increasing the tax rate results in a welfare cost. In any case, the welfare effects are always increasing in the intensity of avoidance \( d \), and the effects of this intensity in the welfare effects are quantitatively important.

5. Concluding remarks

We have introduced a mechanism through which tax avoidance affects economic growth. The growth effects of avoidance emerge in our model from the combination of two forces. On the one hand, individuals can change their ability for avoiding taxes by investing in human capital. In addition, the effective tax rate alters the return from accumulating human capital and the public investment in productive capital. The paper has shown that the interaction between human capital accumulation and tax avoidance may have remarkable growth and welfare effects. Furthermore, these two effects are in general of opposite sign. Avoidance can either increase or reduce economic growth depending on the value of the legal tax rate and on the intensity of the tax avoidance; i.e., the productivity of human capital in avoiding taxes. While the impact of avoidance on the welfare effects of imbalances in human and public capital are small, its impact on the welfare effects of changes in the legal tax rate is quite large. Therefore, the growth-maximizing and the welfare-maximizing tax rates crucially depend on the intensity of avoidance.

The analysis in the paper can be extended in several directions. First, we can perform an optimal taxation analysis. In this type of endogenous growth model, private investment is socially suboptimal because it is a source of productive externalities. The private decision on consumption and investment determines the tax base and thus the stock of public capital and the marginal productivity of human capital. In the present model, the productive externalities also operate through the avoidance technology. A second extension would be to study the effects of avoidance on income inequality. Avoidance may be an important mechanism for the inequality dynamics because it affects tax progressivity. Furthermore, the rate of avoidance differs across the different types of incomes. These two issues will define our research agenda.
References


Table 1. Benchmark economy

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$\rho$</th>
<th>$\sigma$</th>
<th>$\tau$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2499</td>
<td>2/3</td>
<td>0.056</td>
<td>0.005</td>
<td>0.016</td>
<td>2</td>
<td>0.0526</td>
<td>0.0597</td>
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Table 2. Steady-state growth rate $\gamma$

<table>
<thead>
<tr>
<th>Legal tax rate</th>
<th>Intensity of avoidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>tau'd</td>
<td>0</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0051</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0200</td>
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<tr>
<td>0.1</td>
<td>0.0286</td>
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<tr>
<td>0.2</td>
<td>0.0368</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0396</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0392</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0364</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0315</td>
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Table 3. Growth-maximizing avoidance

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$d^*$</th>
<th>$\tau^*$</th>
<th>$\gamma$</th>
<th>$\gamma/0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.26</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>0.26</td>
<td>0.02</td>
<td>0.2562</td>
<td>0.0390</td>
<td>1.9481</td>
</tr>
<tr>
<td>0.30</td>
<td>0.16</td>
<td>0.2646</td>
<td>0.0399</td>
<td>1.9973</td>
</tr>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>0.2820</td>
<td>0.0424</td>
<td>2.1207</td>
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<tr>
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<td>0.54</td>
<td>0.3014</td>
<td>0.0474</td>
<td>2.2444</td>
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<tr>
<td>0.60</td>
<td>0.63</td>
<td>0.3228</td>
<td>0.0449</td>
<td>2.3684</td>
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</table>
Table 4. Growth-maximizing tax rate

<table>
<thead>
<tr>
<th>( d )</th>
<th>( \tau^* )</th>
<th>( \tau^{x/d} )</th>
<th>( \gamma )</th>
<th>( \gamma/0.02 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.33</td>
<td>0.3300</td>
<td>0.0398</td>
<td>1.9884</td>
</tr>
<tr>
<td>0.1</td>
<td>0.38</td>
<td>0.3500</td>
<td>0.0406</td>
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<tr>
<td>0.2</td>
<td>0.43</td>
<td>0.3600</td>
<td>0.0417</td>
<td>2.0825</td>
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<tr>
<td>0.3</td>
<td>0.51</td>
<td>0.4035</td>
<td>0.0430</td>
<td>2.1500</td>
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<td>0.4</td>
<td>0.61</td>
<td>0.4418</td>
<td>0.0449</td>
<td>2.2426</td>
</tr>
<tr>
<td>0.5</td>
<td>0.76</td>
<td>0.5018</td>
<td>0.0475</td>
<td>2.3730</td>
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<tr>
<td>0.6</td>
<td>0.99</td>
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<td>0.0514</td>
<td>2.5710</td>
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</table>

Table 5. Effects of public capital elasticity of output on the growth-maximizing intensity of avoidance

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( 1 - \beta )</th>
<th>( 0.1 )</th>
<th>( 0.25 )</th>
<th>( 1/3 )</th>
<th>( 0.5 )</th>
<th>( 0.7 )</th>
</tr>
</thead>
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<tr>
<td>0.1</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.2</td>
<td>0.63</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.30</td>
<td>0.79</td>
<td>0.36</td>
<td>( 0.16 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.40</td>
<td>0.87</td>
<td>0.55</td>
<td>( 0.40 )</td>
<td>0.22</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.92</td>
<td>0.66</td>
<td>( 0.54 )</td>
<td>0.39</td>
<td>0.42</td>
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Table 6. Effects of public capital elasticity of output on the growth-maximizing tax rate

<table>
<thead>
<tr>
<th>d</th>
<th>0.1</th>
<th>0.25</th>
<th>$\frac{1}{3}$</th>
<th>0.5</th>
<th>0.7</th>
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<tbody>
<tr>
<td>0</td>
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<td>0.33</td>
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<tr>
<td>0.1</td>
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<td>0.38</td>
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<tr>
<td>0.2</td>
<td>0.13</td>
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<td>0.65</td>
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<tr>
<td>0.30</td>
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<td>0.37</td>
<td>0.51</td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td>0.40</td>
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<td>0.61</td>
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<td>0.71</td>
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<td>0.20</td>
<td>0.54</td>
<td>0.76</td>
<td>0.99</td>
<td>0.75</td>
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</tbody>
</table>
Figure 1. Growth effects of avoidance
Figure 2. Growth effects of tax rate
Figure 3. Welfare cost of reducing $h_0$ by a 15% when $\tau = 0.0526$ (benchmark)

Figure 4. Welfare cost of reducing $h_0$ by a 15% when $\tau = 0.4$
Figure 5. Welfare cost of reducing $\tau$ from the benchmark value to 0.4